Motivation

Finnish economy has faced a set of structural changes since the specification and estimation of the latest forecasting model BOF(5/MI

- Substantial decline in labour share,
- Dramatic upward shift in the (average) productivity of capital,
- Dramatic upward shift in profitability and share prices,
- Possibly a speed-up in the rate of technical development.

More is to come
- Demographic changes leading possibly to
- Excess financial burden of pensions and aging population.

Monetary policy should reflect the size of the Finnish Economy relative to Euroarea.

Aino model also wants to build on modern macroeconomic research (DSGE models), and, hopefully, to be more compact and transparent than its predecessors.
Aino

Outline

Organisation and division of labour

- Research project
- How the theoretical model meets data

Basic ingredients of Aino

- Heterogeneous households
- Flexible supply side

Organisation and division of labour

Aino has been organised as a research project:
Aino

- **Consumption, Endogenous Labour Supply, Pension System and Wages in a Life-Cycle Economy** by Mika Kuismanen and Jouko Vilmunen.
- **Firms and Technologies in Aino Model of the Finnish Economy** by Antti Ripatti.
- **Pricing-to-Market and Import Prices in Aino Model** by Antti Ripatti and Hannu Viertola.
- This research has been supervised by Jouko Vilmunen and Juha Tarkka at the Research Department.

Project was hosted by the Research Department January 2001 – April 2002, and then by the Economics Department (as a part-time task)

The components are still to be merged!

**General Structure**

**Households** make the consumption-saving and labour supply decisions. *Aino* has two types of households:

- workers
- pensioners.

Their decisions are influenced by a random transition from work to retirement and a random death probability for pensioners.

The production is performed by five types of **firms**:

- domestic intermediate goods producers,
- retailers:
  - exporter,
  - consumption goods retailer and
  - capital goods retailer,
- the capital rental firm.

Model allows non-unitary elasticity of substitution in the factors of production and **factor specific technical progress**.
Government collects
+ indirect taxes (VAT) and
+ direct taxes on labour and
+ direct taxes on capital
It either
— consumes,
— invests or
— transfers (pensions and other transfers)
its income.

Three assets:
1. domestic one-period bond (public debt),
2. foreign one-period bond (NFAs) and
3. capital (housing and other are not separated).

Short-term interest rates and foreign exchange rates are exogenously given.

How the theoretical model meets data

We deviate from the Cobb-Douglas world of constant factor shares. Therefore we are able specify factor specific technology and preference terms.

Due to this reason our model is rich of stochastic ‘shocks’. These shocks are well defined in the sense we can label them.

These technology and preference terms are unobservable and time-varying.

Given the parameter (gu)estimates and data, we may recover these unobservables and tell theory-consistent story based on the recovered unobservables. This is the way we identify shocks. We obtain perfect fit!

In some sense these technology and preference terms act as add-factors. The novelty is that they have label, hence, they tell a story.

In forecasting one may use whatever time-series or expert methods to forecast these animals.

Market Structure of Aino Model
Market Structure of *Aino* Model
In the near future demographic changes will affect to the macroeconomic environment through the many different channels:

- growth rate of human capital,
- wealth distribution,
- marginal propensity to consume,
- public transfers.

Traditional
- over-lapping-generation models,
- growth models with homogenous population assumptions,
- Blanchard-Yaari framework with homogenous population assumption,

are not applicable to study life-cycle behaviour. Because individuals currently alive are identical\(^1\). They have identical marginal propensities to consume.

\(^1\)Except for their respective levels of human wealth in the Blanchard–Yaari approach.
It is, therefore, not possible to study the impact of policies that redistribute wealth between workers and pensioners, and it is also impossible to study demographic changes.

Our approach is to extend Gertler’s (1999) new kind of overlapping generations model to analyse the economic impacts of consumption and endogenous labour supply. In this framework individuals exhibit life-cycle behaviour and, in addition, are allowed to have realistic average lengths of life, work and retirement.

Adding life-cycle factors is likely to enhance the impact of government debt and deficits for the following reasons:

- Allowing a retirement period raises the fraction of government bonds that are net wealth to those currently alive because it shortens the horizon over which the current workforce is liable for future taxes.
- Having pensioners and workers implies that a rise in government debt will redistribute wealth from a low propensity to consume group (workers) to a high propensity to consume one (pensioners).

**Pensions**

**NOTE:** The implementation of the statutory earnings-related pension provision, i.e. collection of contributions, management of funds and payment of pensions is handled by private pension institutions. However, they operate according to common principles. The earnings-related pension cover for the insured is always protected against possible insolvency of the pension institutions. The pension institutions are jointly and severally liable for payment of pensions in such cases.

The administration of the Finnish statutory earnings-related pension scheme is decentralised. Most employers and self-employed persons may choose their own pension institution. The pension institution may be a pension insurance company, an industry-wide pension fund or a company pension fund. In addition, there are also some pension institutions for special branches of industry.

Finnish pension system is partly (approx. 20 per cent) funded. Otherwise it is pay-as-you-go (PAYG) system. It is decentralised. Pension funds are private but they are jointly liable for payment of pensions (e.g. in the case of insolvency). Beneficiaries may not influence the management of funds or amount of savings. The contribution rates, etc., are administrated. These funds are classified as a part of general government in national accounts.

Besides the statutory earnings-related pension scheme there is also a national pension scheme covering all citizens and increasingly popular non-statutory pension schemes, which are partly tax deductible.
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From the modelling point of view, we consider the funded part of the pension system as normal private savings and the PAYG part as a transfer from workers to pensioners. Hence, the pensions — when mentioned in the theoretical model — are simple transfers and do not correspond the actual amount of money received by pensioners.

Setting Up the Model

Each individual is born as a worker. Conditional on being a worker in the current period the probability of remaining as a worker in the next period is χ and thus the probability of retiring is 1 − χ.

The probability of surviving to the next period is κ conditional of being retired in the previous period and, thus, the probability of death in the next period is 1 − κ.

NOTE: Above formulation is just a generalization of the standard two period overlapping generation model. If χ = 0 and κ = 0, individual works in the first period of life and retires (with certainty) and the dies with certainty at the end of the second period.

Population, N_t, growth: (1 − χ + n)N_t new workers are born in t + 1 implying that the workforce grows at the gross rate 1 + n.

Ratio of retirees to workers is

\[ \psi = \frac{1 - \chi}{1 + n - \kappa} \]

Assumptions

A We assume aggregate risk to be nonexistent. The only risks are are idiosyncratic.

NOTE: Worker faces a potential loss of wage income and retiree faces an uncertain time of death.

B Perfect annuities markets are introduced to eliminate the impact of uncertainty about time of death.

NOTE: Retiree give his wealth to a mutual fund which in turn invest them. The fraction of κ of those who survive to the next period receive all returns, while the fraction of 1 − κ who die receive nothing. Each surviving retiree receives a return that is proportional to his contribution to the fund. If R is the gross return per unit invested by the fund then the gross return for a surviving retiree is \( \frac{R}{\kappa} \).
C Individual’s preferences are restricted to address the risk of uncertainty of retirement. **NOTE:** To restrict individuals to be risk neutral respect to income risk, but allow for an arbitrary intertemporal elasticity of substitution. Since the degree of income risk is artificial in the sense that it is generated by the assumption of constant probability of transition into retirement, it seems reasonable to mitigate this the impact of this income variation by assuming risk neutrality.

D Individuals are risk neutral with respect to income risk, but have arbitrary intertemporal elasticity of substitution, \( \sigma \equiv 1/(1 - \rho) \):

\[
V_t^i = \left\{ \left[ (C_t^i)^\nu \left( 1 - l_t^i \right)^{1-\nu} \right]^\rho + \beta^i E_t(V_{t+1}^i)^{\rho} \right\} \quad i = \{\text{worker } (w), \text{pensioner } (p)\}
\]

with \( \beta^w = \beta \) and \( \beta^p = \beta \kappa \). \( C_t \) is consumption basket, \( V_t^i \) is individual’s value function and \( \beta \) is the subjective discount factor. **NOTE:** Since the degree of income risk is artificial in the sense that it is generated by the assumption of constant probability of transition into retirement, it seems reasonable to mitigate this the impact of this income variation by assuming risk neutrality.

### Consumption and Labour Supply Decision

Notation \((i = \{w, p\})\):

- \( l_t^w \): fraction of time allocated to work
- \( \xi \in (0, 1) \): relative labour ‘productivity’ of a pensioner
- \( A_t^i \): Assets
- \( C_t^i \): Consumption
- \( R_t \): Gross return on assets
- \( W_t \): Market wage
- \( \lambda_t^P \): Pension contribution
- \( \lambda_t^P = \lambda_t^P W_t \): Pensions (non-funded part)
- \( L_t^i \): Labour tax
- \( u_t \): Probability of being unemployed
- \( \eta_t \): Replacement ratio
- \( \epsilon_t \): Ratio of pensioner’s MPCW to worker’s MPCW
- \( \pi_t \): Worker’s MPCW

**Worker**

Conditional expectation of the value function

\[
E_t \left( V_{t+1}^w \right) = \chi V_{t+1}^w + (1 - \chi) V_{t+1}^p.
\]

**Pensioner**

Conditional expectation of the value function

\[
E_t \left( V_{t+1}^p \right) = V_{t+1}^p.
\]
Aino

**Worker**

Period-by-period budget constraint:

\[
A_{t+1}^w = R_t A_t^w + [(1-u_t)(1-t_t^L-t_t^P)] W_t \\
+ u_t (1-t_t^L-t_t^P) \eta_t W_t] l_t^w - C_t^w \\
= R_t A_t^w + [1 - (1-\eta_t)u_t] (1-t_t^L-t_t^P) \\
\equiv m_t \\
\times W_t l_t^w - C_t^w
\]

Intra-period labour supply:

\[
l_t^w = 1 - \frac{(1-\nu)/\nu}{m_t W_t} C_t^w,
\]

**Pensioner**

Period-by-period budget constraint:

\[
A_{t+1}^P = \frac{R_t}{\kappa} A_t^P + (1-t_t^L) \xi_t^P + O_t^P - C_t^P.
\]

Intra-period labour supply:

\[
l_t^P = 1 - \frac{(1-\nu)/\nu}{\xi(1-t_t^L)W_t} C_t^P,
\]

and Euler equation of consumption

\[
C_t^P = \left\{ \left[ \frac{(1-t_t^L)W_t}{(1-t_{t+1}^L)W_{t+1}} \right]^{(1-\nu)\rho} \beta R_t \right\}^\sigma C_t^P
\]
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Worker’s Euler equation of consumption

\[
\chi C_{t+1}^w + (1 - \chi) (\epsilon_{t+1})^{-1/\rho} \left( \frac{m_{t+1}}{\xi (1 - t_{t+1}^L)} \right)^{1-\nu} C_{t+1}^p = \left[ \frac{m_t W_t}{m_{t+1} W_{t+1}} \right]^{(1-\nu)\rho} \beta R_t \Omega_{t+1}
\]

where

\[
\Omega_t = \chi + (1 - \chi) (\epsilon_t)^{1/(1-\sigma)} \left( \frac{m_t}{\xi (1 - t_t^L)} \right)^{1-\nu}
\]

The marginal propensity to consume (worker \( \pi_t \); pensioner \( \epsilon_t \)) out of wealth

\[
\pi_t = \nu \left[ \frac{1}{1 - \sigma} \left( \frac{m_t W_t}{m_{t+1} W_{t+1}} \right)^{(1-\nu)\rho} \left( R_t \Omega_{t+1} \right)^{\sigma-1} \right]
\]

\[
\epsilon_t \pi_t = \nu \left[ 1 - \left( \frac{(1 - t_t^L) W_t}{(1 - t_{t+1}^L) W_{t+1}} \right)^{(1-\nu)\rho} \sigma \right] \times \beta R_t \left( \frac{1}{\epsilon_{t+1}} \right) ^{t_{t+1}^p} \left( \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}} \right)
\]

It can be shown that \( \epsilon_t > 1 \) always!

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**Parameter Calibrations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9725</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>( \chi )</td>
<td>0.99407</td>
<td>Average working time is 42 years</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.98076</td>
<td>Average pension time is 13 years</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.105</td>
<td>Efficiency of pensioners as workers (to calibrate pensioners’ labour supply)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.75</td>
<td>Budget share of consumption</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>( \frac{1}{1-\rho} )</td>
<td>0.23</td>
</tr>
<tr>
<td>( n )</td>
<td>0.001</td>
<td>Population growth rate (0.4 % per annum)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>1.756</td>
<td>Ratio of retirees to workers</td>
</tr>
<tr>
<td>( t_P )</td>
<td>0.05</td>
<td>Pension contribution</td>
</tr>
<tr>
<td>( t^L )</td>
<td>0.3</td>
<td>Average labour tax</td>
</tr>
</tbody>
</table>

---

**Aggregation Issues**
Marginal propensity to consume out of wealth is same for all the pensioners. \( A^P_t \) is the total financial assets that has the aggregate return \( R_t \). Workers similarly.

 Aggregate Consumption function:

\[
C_t = \pi_t \left\{ [1 + (\epsilon_t - 1)\lambda_t]R_tA_t + \epsilon_tN^P_t(H^P_t + S^P_t) + N^u_t(H^u_t - S^u_t) \right\},
\]

where \( \lambda_t \equiv A^P_t/A_t \) is financial wealth held by pensioners,

Since \( \epsilon_t \) exceeds unity, a rise in \( \lambda_t \) raises aggregate consumption.

\[ \implies \text{pension transfers will influence aggregate consumption and labour supply.} \]

Because effective discount rate used to measure aggregate human wealth exceeds the market interest rate, government debt and deficits will influence behaviour.

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**Market Structure of *Aino* Model**

![Market Structure Diagram]

- Capital rental firm
- Domestic intermediate goods producer
- Aggregate
- Consumption and labour supply
- Investment goods retailer
- Exporter
- Consumption goods retailer
- Importing firm
- Importing firm
- Importing firm

Labour Union

\( K \)
Wage formation is based on the right-to-manage model, where labour unions maximises utility gained from workers’ wage and unemployment benefit income subject to the demand for labour.

Rigidities arise from Calvo contracts.

Market Structure of Aino Model

Domestic intermediate goods, $Y_t(j)$ are produced by the continuum of firms who face monopolistic competition. They are aggregated to the composite good by the
following aggregator

\[ Y_t = \left[ \int_0^1 Y_t(j) - \rho_t^2 \, d \frac{1}{\rho_t^2} \right]. \]

The parameter \( \rho_t^2 \in [-1, \infty) \) determines the elasticity of substitution \( 1/(1 + \rho_t^2) \). It also defines the time-varying mark-up. Its time-path is exogenous.

The cost minimization implies the following demand function for the individual goods

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\frac{1}{1+\rho_t^2}} Y_t. \]  

(1)

**Market Structure of Aino Model**

![Diagram of market structure](image)

**Domestic Intermediate Goods Producers**
Domestic intermediate good, $Y_t(j)$, is produced by producers who face monopolistic competition. They take the production technology and the factor augmenting technical trends as exogenously given. The production function is of CES type and assumes constant-returns-to-scale.

\[
Y_t(j) = \left[ \delta \left( \Lambda_t^K K_t(j) \right)^{-\rho} + (1 - \delta) \left( \Lambda_t^L L_t(j) \right)^{-\rho} \right]^{-1/\rho},
\]

where
- homogenous capital services, $K_t(j)$, and
- non-homogenous labour, $L_t(j)$;
- capital-augmenting technical progress $\Lambda_t^K$ and
- labour-augmenting technical progress $\Lambda_t^L$,
- $\delta$ refers to share parameter.

**Prices**

The cost minimization implies the following real marginal costs

\[
\frac{MC_t(j)}{P_t(j)} = \left[ \delta \left( \frac{R_t}{\Lambda_t^K P_t(j)} \right)^{\frac{\rho}{1+\rho}} + (1 - \delta) \left( \frac{W_t}{\Lambda_t^L P_t(j)} \right)^{\frac{\rho}{1+\rho}} \right]^{\frac{1+\rho}{\rho}},
\]

where
- $R_t$ denotes the nominal rental price of capital services and
- $W_t$ nominal wages.

In steady-state, prices, $P(j)$, are determined by mark-up, $\Upsilon$ over marginal costs

\[
P(j) = \Upsilon MC(j), \quad \text{where mark-up is given by } \Upsilon = \frac{1}{\rho^z}.
\]
The dynamics of the price level $P_t(j)$ of producer $j$ arises from the assumption that a firm changes its price level when it receives a random “price-change signal”. Probability of receiving a price change signal is given by $1 - \zeta$ ($\zeta \in [0, 1]$). It is constant. Since there is continuum of intermediate producers, $1 - \zeta$ also represents the share of producers that has received such a signal and, consequently, got an opportunity to change their prices. The average time between price changes is given by $1/(1 - \zeta)$.

Assuming symmetry of the firms and linearising the first order condition and equation of motion of the aggregate price level we obtain the following Phillips curve:

$$\Delta p_t = M E_t \Delta p_{t+1} + \frac{(1 - \zeta)(1 - \zeta M)}{\zeta} [v_t + mc_t - p_t],$$

where $M$ is the steady-state level of the discount factor. Note that the time-varying mark-up $v_t$ affects inflation.

### Market Structure of *Aino* Model

![Market Structure Diagram](image-url)
Capital Rental Firm

Capital is homogenous factor of production that is owned by firms that rent capital to the producers of domestic intermediate goods. It operates under perfect competition.

The capital rental firms may choose between physical capital accumulation $K_t^P$ or a higher utilization rate $U_t$ with $K_t = U_t K_{t-1}^P$ ($U_t \in [0, 1]$).

Physical accumulation generates real adjustment costs in the form of lost capital stock, whereas the capital utilization rate affects the depreciation of the capital stock.

$$K_t^P + \frac{\gamma_1}{2} \left( \frac{\Delta K_t^P - \gamma_2 \Delta K_t^P}{K_{t-1}^P} \right)^2 = K_{t-1}^P \left[ 1 - \left( \delta_0 + \frac{\delta_1}{1 + \delta_2 U_t^{1+\delta_2}} \right) \right] + I_t,$$

The capital rental firm maximizes its expected discounted profits

$$\max_{\{U_t\}, \{I_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} M_{t,t+s} K_{t+s}$$

subject to capital accumulation equation and the definition of capital services. The momentary profits are given by

$$\Pi_{t} = R_t K_t - P_t I_t$$

$$= R_t U_t K_{t-1}^P - P_t I_t \left\{ K_t^P + a K(K_t^P, K_{t-1}^P, K_{t-2}^P) - K_{t-1}^P [1 - D(U_t)] \right\}.$$

The price index of investment goods is the price index of the domestic investment good retailer, $P_t^I$.

Since the firm is owned by households, the future profits are discounted using the nominal stochastic discount factor (pricing kernel) $M_{t,t+s} = \beta^s U'(C_{t+s}) P_t^C / [U'(C_t) P_{t+s}^C]$ ($P_t^C$ refers to price index of composite consumer goods).

Market Structure of Aino Model
Estimates of the parameters related to the ‘investment function’

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu^g$</td>
<td>-19.37</td>
<td>26.24</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>0.978</td>
<td>1.70</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>55.47</td>
<td>198.07</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>1.47</td>
<td>24.41</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>13.29</td>
<td>120.46</td>
</tr>
<tr>
<td>$\delta^g$</td>
<td>-0.25</td>
<td>88.63</td>
</tr>
<tr>
<td>$\gamma^g$</td>
<td>7.80</td>
<td>384.43</td>
</tr>
<tr>
<td>$\tau^g$</td>
<td>0.75</td>
<td>119.35</td>
</tr>
</tbody>
</table>

Significance level of $J$-test: 0.39.
**Aino**

<table>
<thead>
<tr>
<th>Instrument</th>
<th>lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross operating surplus relative to nominal GDP</td>
<td>0, 1</td>
</tr>
<tr>
<td>Log-difference of capital stock</td>
<td>2,3</td>
</tr>
<tr>
<td>Investment deflator</td>
<td>1</td>
</tr>
<tr>
<td>Empirical stochastic discount factor</td>
<td>1</td>
</tr>
<tr>
<td>Effective share yield</td>
<td>0</td>
</tr>
<tr>
<td>Average productivity of capital</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Derived parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.0125</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.010</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.034</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>13.29</td>
</tr>
</tbody>
</table>

**Estimates of Unobservables**

![Graphs showing estimated values over time](image)

$D(U)$
Figure 1:

$D(U)$

Market Structure of *Aino* Model
Retailers

The economy is inhabited by two retailers. One is specialized for consumer goods and the other one for capital goods. They operate under perfect competition. They combine

- Domestic intermediate goods ($Y^C_t$ and $Y^I_t$ respectively)
- Imported goods and services ($M^C_t$ and $M^I_t$ respectively)

with the CES aggregator

$$C^T_t = \left[ \delta^C \left( \Lambda^C Y^C_t \right)^{-\rho^C} + (1 - \delta^C) \left( \Lambda^C M^C_t \right)^{-\rho^C} \right]^{-1/\rho^C}$$

Capital goods respectively. Elasticity of substitution is given by $1/(1 - \rho^C)$.

The factor-augmenting technical changes $\Lambda^C Y$ and $\Lambda^C M$ model preference shifts.

The output of the retailers are consumed (invested) both households and public sector. Indirect taxes, $t^IN_t$, are levied on output of the consumption goods retailer.
Estimation of the elasticity of substitution is based on the first order condition wrt imported factor.

**Exporter**

Exporter’s case is analytically identical to that of the retailers.

Each of the exporting firm is price-taker, ie makes no profits. As an aggregate, the Finnish exporting firms face the following demand function

\[ X_t = \left( \frac{P_t^X}{P_t^F} \right)^{-\rho} W \]

*World Demand*

**Parameters of Retailers and Exporter**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Consumption goods retailer</th>
<th>Investment goods retailer</th>
<th>Exporter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>-0.773</td>
<td>-0.538</td>
<td>1.22</td>
</tr>
<tr>
<td>standar error</td>
<td>0.049</td>
<td>0.183</td>
<td>0.38</td>
</tr>
<tr>
<td>( 1/(1+\rho) )</td>
<td>4.4</td>
<td>2.2</td>
<td>0.45</td>
</tr>
<tr>
<td>Calibr. ( \delta )</td>
<td>0.87</td>
<td>0.67</td>
<td>0.51</td>
</tr>
</tbody>
</table>

It seems that the estimated elasticity of substitution of consumption goods retailer is far too high.

**Market Structure of Aino Model**
Model has three importing firms — one for each type of imported goods — who operate outside Finnish borders, and, therefore, do not influence to the accounting scheme of the model.

For each type of imported goods, it is assumed that a fraction $\omega$ of these firms price their products in the Finnish currency (local currency pricing) and $1 - \omega$ price their products in their own currency (producer currency pricing). Both type of firms share the same (constant-return-to-scale) production technology, ie their marginal costs are identical.

They operate under imperfect competition. They face random price change signal, $\zeta$, ie Calvo-frictions are applied here. $R^*$ is their discount factor.
Facing demand functions

\[
M^F(j) = \left[ \frac{P^F(j)}{P} \right]^{\frac{1}{\frac{1}{1+\rho}}} M
\]

and momentary profits

\[
\Pi^F_{t+k}[P^F(j)] = P^F_j M^F_{t+k}(j) - S_{t+k} C_{t+k}(j) M^F_{t+k}(j)
\]

Calvo pricing

\[
\Delta p^F_t = R^* E_t \Delta p^F_{t+1} + \frac{(1-\zeta)(1-\zeta R^*)}{\zeta} (s_t + mc_t - p^F_t)
\]

Aggregation according to

\[
p_t = \omega p^F_t + (1-\omega) p^P_t
\]

Final form

\[
\Delta p_t = R^* E_t \Delta p_{t+1} + \frac{(1-\zeta)(1-\zeta R^*)}{\zeta} (s_t + mc_t - p_t)
\]

\[+(1-\omega)(\Delta s_t - R^* E_t \Delta s_{t+1})\]

Estimates for the three import-price equations

<table>
<thead>
<tr>
<th>Sub-group</th>
<th>$R^*$</th>
<th>$\zeta$</th>
<th>$\omega$</th>
<th>$J$-test</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer goods</td>
<td>0.516</td>
<td>0.889</td>
<td>0.831</td>
<td>9.319</td>
<td>0.068</td>
</tr>
<tr>
<td>Std. error</td>
<td>0.196</td>
<td>0.022</td>
<td>0.031</td>
<td>(0.316)</td>
<td>0.028</td>
</tr>
<tr>
<td>Capital goods</td>
<td>0.492</td>
<td>0.896</td>
<td>0.939</td>
<td>9.244</td>
<td>0.065</td>
</tr>
<tr>
<td>Std. error</td>
<td>0.184</td>
<td>0.032</td>
<td>0.029</td>
<td>(0.322)</td>
<td>0.023</td>
</tr>
<tr>
<td>Materials</td>
<td>0.624</td>
<td>0.953</td>
<td>0.903</td>
<td>10.985</td>
<td>0.020</td>
</tr>
<tr>
<td>Std. error</td>
<td>0.096</td>
<td>0.024</td>
<td>0.077</td>
<td>(0.203)</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Estimated using GMM. Instruments include: constant, lags of import volumes (varies
between sub-groups), import prices of various sub-groups and its lag, private consumption/investments/export prices and its lag, public consumption/investment prices and its lag.

Poor estimates are, probably, due to the poor (data) estimates of the marginal costs, i.e., international price indices.

Production of Public Goods

Public consumption $C_t^G$ is divided into two components

- market goods, $C_t^{GM}$, and
- non-market goods, $C_t^{GY}$.

Market goods are produced by the consumption goods retailer and non-market goods by the public sector itself using the following production technology

$$C_t^{GY} = \Lambda_t^G \left( L_t^G \right)^{\delta^G},$$

where $\Lambda_t^G$ is the technological trend, and $L_t^G$ public employment. Parameter estimate of $\delta^G$ is close to unity (0.9).

Market Structure of Aino Model
References